

Data Structures and Algorithms COMP-251 A

Problem Assignment #1

1. Induction Proof (arrangement of lines)

Consider n lines ($n > 2$) in general position in the plane. Prove by induction that at least one of the regions they form is a triangle.

2. Induction Proof (number theory)

Find an expression, as a function of n , of the sum of the squares of the first n natural numbers and then prove your claim by induction.

3. Induction Proof (circle map coloring)

Prove by induction that the regions formed by n circles in the plane can be colored with two colors such that any neighboring regions are colored differently.

4. The *Collapsing Compass Computer*

In proposition II of *Book I* of *The Elements*, Euclid proposed an algorithm that takes nine steps (using the *straight-edge-and-collapsing-compass* computer) to *transfer* (or displace) a given line segment $[a,b]$ to a new location so that one end point, say a , lies on a pre-specified point c in the plane. The transferred segment obtained by executing Euclid's algorithm is not necessarily *parallel* to the original segment (i.e., the *displacement* is not necessarily a *translation*).

(a) Design an algorithm that uses the same computer (the *straight-edge-and-collapsing-compass*) and necessarily yields a translation. The fewer the steps the higher the mark. The steps we are counting here are the number of times we use the straight edge to draw a line plus the number of times we use the collapsing compass to draw a circular arc or circle.

(b) Prove that your algorithm in (a) is correct.