The Rhythm that Conquered the World: What Makes a “Good” Rhythm Good?

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Dedicated to the memory of Bo Diddley.

THE *BO DIDDLEY* BEAT

Much of the world’s traditional and contemporary music makes use of characteristic rhythms called *timelines*. A timeline is a distinguished rhythmic ostinato, a rhythm that repeats throughout a piece of music with no variation, that gives the particular flavor of movement of the piece that incorporates it, and that acts as a timekeeper and structuring device for the musicians. Timelines may be played with any musical instrument, although a percussion instrument is usually preferred. Timelines may be clapped with the hands, as in the flamenco music of Southern Spain, they may be slapped on the thighs as was done by Buddy Holly’s drummer Jerry Allison in the hit song “*Everyday*”, or they may just be felt rather than sounded. In Sub-Saharan African music, timelines are usually played using an iron bell such as the *gankogui* or with two metal blades. Perhaps the most quintessential timeline is what most people familiar with rockabilly music dub the *Bo Diddley Beat*, and salsa dancers call the *clave son*, which they attribute to much other Cuban and Latin American music. This rhythm is illustrated in box notation in Figure 1. Each box represents a pulse, and the duration between any two adjacent pulses is one unit of time. There are a total of sixteen pulses, the first pulse occurs at time zero and the sixteenth at time 15. An empty box denotes a silent pulse or rest, and a box filled with a mark indicates a sounded (or felt) pulse. Thus the sounded pulses, also called *onsets*, for the clave son are those numbered 0, 3, 6, 10, and 12. This rhythm may also be represented by its sequence of adjacent inter-onset intervals (IOI’s), that is [3-3-4-2-4].

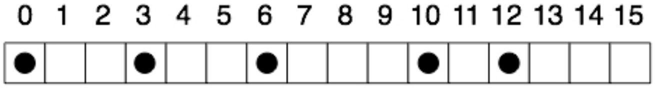


Fig. 1: The *clave son* in box notation.

In Afro-Cuban music the clave son is played using a pair of sticks called *claves*, usually made of hard wood that produces a crisp captivating tone that cuts through the variety of different sounds produced by all the other instruments being played. Since a timeline is continually repeated, and thus cyclic, it is often convenient to represent it by a set of points on a circle, sometimes called a *clock diagram*. Figure 2 shows the clave son represented as a polygon on such a clock diagram. The cycle contains the sixteen equally spaced points (pulses) indicated by small circles. The white circles correspond to silent pulses, whereas the black filled circles denote sounded pulses. The rhythm begins on the pulse labeled zero, time flows in a clockwise direction, and the distance along the circle (arc) between two adjacent pulses corresponds to one unit of time. Connecting the adjacent onsets of a rhythm with straight-line segments, shown in Figure 2, yields yet another representation of the rhythm as a convex-polygon. Such a representation, it will be seen, is quite useful for a variety of different types of analyses.

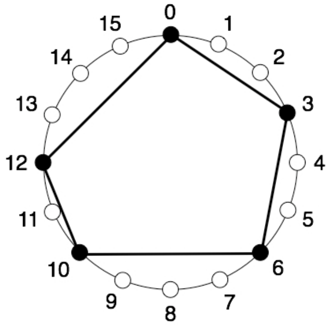


Fig. 2: The *clave son* in clock-diagram and polygon notations.

Very little is known about the history and evolution of this rhythm. Although most people associate it with the Cuban son and its offspring that include salsa and rockabilly, we can add a few additional historical facts that go into the much further distant past. In Ghana the rhythm is played on an iron bell and called the *kpanlogo* timeline. Some ethnomusicologists believe that this rhythm was transported from West Africa to Cuba with the slave trade sometime during the past five hundred years. Others believe that another similar ancestral 12-pulse (ternary) rhythm with inter-onset interval sequence [2-2-3-2-3] travelled from West Africa to Cuba, where, with the influence of Spanish music, it was transformed or mutated into the binary16-pulse version of the rhythm. It is difficult to be sure of the rhythm's historical trajectory, since it was subject to an oral tradition in Sub-Saharan Africa. However, there exists a fascinating manuscript written in Bagdad in the middle of the Thirteenth Century by the music scholar Safi al-Din, titled "Kitāb al-Adwār," in which this rhythm is notated and labeled "*al-thaqil al-awwal*." Safi al-Din was one of the lucky survivors of the almost complete destruction of Bagdad by the Mongol invasion of 1258, and it makes one wonder if playing this rhythm for his captors contributed to saving his life. Presently it is not known if this rhythm migrated from Bagdad to West Africa or vice versa, or whether it was born in both places independently, and to the author's knowledge, no earlier written record of this rhythm exists. Today the clave son enjoys the reputation as the most popular rhythm on the planet. It is heard in all corners of the world, in almost any type of music, including rhythm and blues, salsa, rockabilly, rock, soukous, jazz, house, and the fusion pop music of scores of countries. Indeed, it is fair to say that this is the rhythm that has conquered the world. For this reason, this 16-element binary sequence, this innocent-looking little pattern with inter-onset intervals [3-3-4-2-4] is a precious cultural object of great significance, and thus worthy of study. Several natural questions arise about a pattern that ascends to the throne of musical timelines. What makes this rhythm so special? How can we explain its saliency as a timeline, and its seductive power over the human ear, and why did humanity come to prefer this rhythm above all others? This article provides answers to these questions.

MAXIMALLY EVEN RHYTHMS

From a purely combinatorial point of view the rhythm in Figure 1 is merely one way of placing five pigeons into sixteen pigeonholes such that no pigeonhole has more than one pigeon. It is a high-school arithmetical exercise to calculate that there are 4,367 other ways of doing this! What is so special about this one particular configuration? Before turning to this issue, there are two other preliminary questions that should be dispensed with first: what is so attractive about a sequence of *sixteen* pulses? Why not eleven, thirteen, or seventeen for example? And what is it that is so singular about *five* onsets? Why not four, six, or nine? These two numbers, the number of pulses in the cycle of a timeline, and the number of these pulses that are sounded, vary widely among different cultures around the world. It is quite common for the number of pulses in the cycle to be as little as four. In Bulgarian music it may go as high as 33, and in the *talas* of Indian classical art music it may be as long as 128. The answers to these questions are essentially physiological and psychological; they lie to a large extent in the nature of the mental and physical constraints imposed by the human brain and body. Fundamentally, to be popular a rhythm should not be so complex that it becomes difficult to grasp by the masses, and at the same time it should not be so simple that it quickly becomes boring. Furthermore, to serve well as a timeline for dancing, its realization should not take much more than about two seconds, the duration of our conscious sense of the present. Rhythms with an even number of pulses that is also a power of two are for most people of the world, easier to assimilate than other rhythms. These constraints are already sufficient to bring the workable number of pulses down to small values that are powers of two, such as eight or sixteen. As for the number of onsets, for a timeline to afford a rich enough structure, five appears to be a good choice. However, a cycle of eight pulses does not provide enough room (in the sense of time) for five onsets to be distributed so as to create interesting patterns. If only three onsets are required then eight pulses are sufficient to create a viable timeline, namely [3-3-2], the first half of the clave son (called the *tresillo* in Cuba), and itself one that also enjoys world popularity. Thus we are left with sixteen pulses and five onsets as the most feasible candidates for creating a timeline that has a sufficiently rich structure.

The number of different rhythms that may be created with five onsets and sixteen pulses is still a staggering 4,368. One of the most important desirable properties that a timeline should possess, and that will eliminate almost all the candidates from the competition, is that the onsets should be distributed among the pulses *almost* as *evenly* as possible. Distributing four onsets as evenly as possible among sixteen pulses presents no problem: the solution is a rhythm with interval sequence [4-4-4-4]. On the other hand this pattern would be a boring and endless isochronous sequence of sounds that most musicologists would not even include in their definition of rhythm. The number five on the other hand does not divide evenly into sixteen; it gives a value of 3.2. A nice way to visualize this division and the resulting recipe for generating the almost maximally even rhythms with five onsets among sixteen pulses is illustrated in Figure 3, where the horizontal axis indicates the pulse number as a function of time, and the vertical axis indicates the index of the five onsets. The diagonal line connecting the first onset at pulse zero on the lower left, to its cyclic self-image, on the upper right, creates with the horizontal lines for each onset a set of intersection points that indicate the times at which the five perfectly evenly distributed onsets should be played: 0.0, 3.2, 6.4, 9.6, and 12.8. Each adjacent pair of onsets would thus be 3.2 units of time apart. But this would produce no rhythm at all, just an isochronous pulse. The onsets must be played at the times at which the pulses occur, marked in Figure 3 with pairs of black dots.

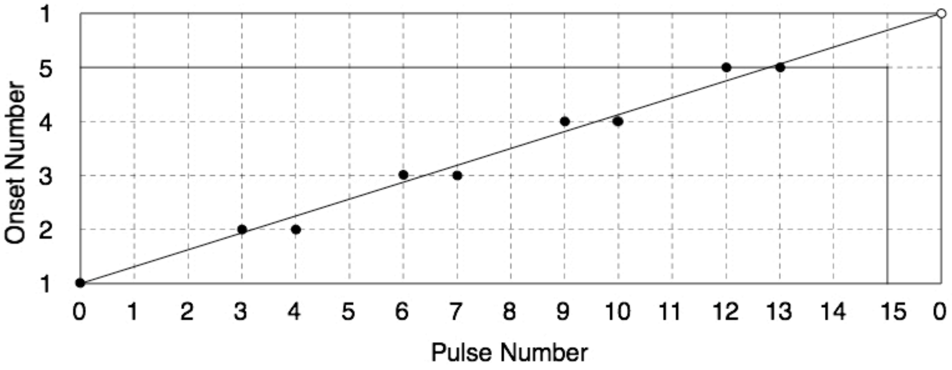
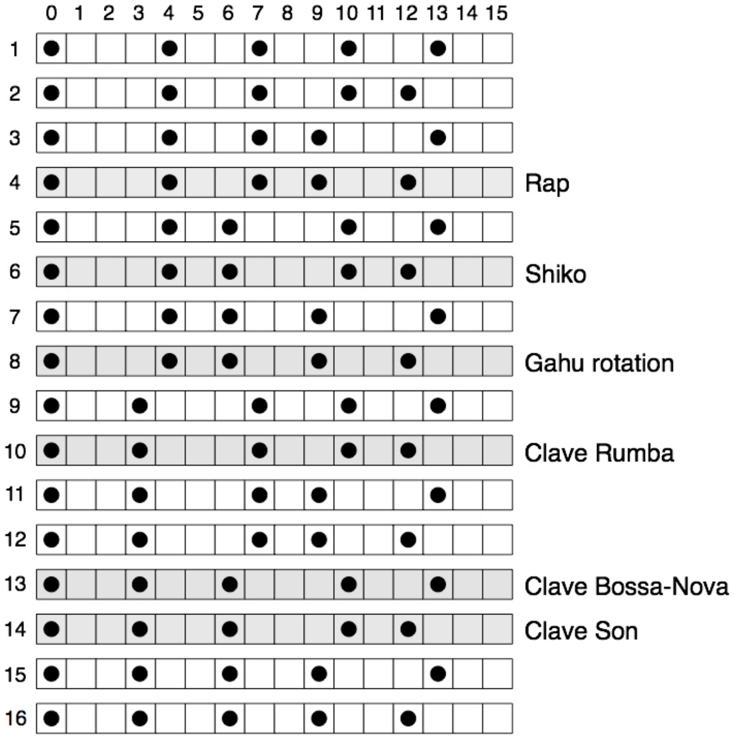


Fig. 3: Generating the sixteen almost maximally even 5-onset, 16-pulse rhythms.

An optimal maximally even rhythm may be obtained by snapping the intersection points of the horizontal lines with the diagonal to their *nearest* pulse. Thus 3.2 would snap to 3, 6.4 to 6, 9.6 to 10, and 12.8 to 13, yielding the rhythm [3-3-4-3-3]. The *almost maximally even* rhythms are defined as all the rhythms with five onsets made up of one onset at pulse 0, and one onset from each pair of black dots on either side of the diagonal line. These onsets are obtained by snapping each intersection point to either its *nearest left* pulse *or* its *nearest right* pulse. Therefore at each of the four onsets following the first onset there are two onsets to choose from, and the total number of almost maximally even rhythms is 2×2×2×2 = 16. These sixteen rhythms are pictured in Figure 4, where they are listed in lexicographical order.



	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	●				●			●			●			●			
2	●				●			●			●		●				
3	●				●			●		●				●			
4	●				●			●		●			●				Rap
5	●				●		●				●			●			
6	●				●		●				●		●				Shiko
7	●				●		●			●				●			
8	●				●		●			●			●				Gahu rotation
9	●			●				●			●			●			
10	●			●				●			●		●				Clave Rumba
11	●			●				●		●				●			
12	●			●				●		●			●				
13	●			●			●				●			●			Clave Bossa-Nova
14	●			●			●				●		●				Clave Son
15	●			●			●			●				●			
16	●			●			●			●			●				

Fig. 4: The sixteen almost maximally even rhythms with 5 onsets among16 pulses.

Note that in addition to the clave son, which is member number 14 of this family of rhythms, there are several other noteworthy rhythms (highlighted) that are used as timelines in different genres of music. Rhythm number 4 is a common timeline used in rap music, and was employed by Chucho Valdez as a drum timeline in his jazz composition *Invitation*, released by EGREM CD0233 in Havana, Cuba, in 1997. Rhythm number 6 is common in many cultures and sometimes called the shiko bell pattern. Because all its inter-onset durations are multiples of two it may be expressed as the 8-pulse pattern [2-1-2-1-2], and is also referred to as one of the *cinquillo* patterns in Cuba. Rhythm number 8 is a rotation of the bell pattern used in the Gahu drumming music of West Africa. The actual timeline is started on the third onset of rhythm number 8. Rhythm number 10 is the clave timeline used extensively in the rumba style of Cuban music. Rhythm number 13 is the timeline used in the bossa-nova music of Brazil. Note that several other rhythms are rotations of each other: rhythms 1, 9, 15, and 16 are rotations of the bossa-nova, rhythm 3 is a rotation of the rumba, and rhythms 5 and 11 are rotations of the clave son.

From the preceding exposition it follows that imposing the requirement that 5-onset, 16-pulse rhythms should be almost maximally even greatly reduces the number of potential candidates for good timelines from 4,368 to 16. To reduce this drastically shorter list of 16 down to one requires recruiting a few more desirable properties that a good timeline should possess.

THE RHYTHMIC ODDITY PROPERTY

Compare the conga rhythm in Figure 5 (left) with the clave son on the right. The Figure shows a diameter of the circle emerging from each onset in the rhythms to its diametrically opposite (antipodal) pulse. The conga rhythm has two onsets, at pulses zero and eight, that are located diametrically opposite each other. They divide the cycle into two half-cycles (of equal duration). Regular rhythms with an even number of onsets contain many such pairs of antipodal onsets, and in general, the presence of such pairs contributes to making the rhythm less exciting than it otherwise could be. It is generally considered that syncopation is the spice of rhythm. Almost maximally even rhythms that do not possess antipodal pairs of onsets will tend to be more syncopated. The clave son on the right, for example, contains no pairs of antipodal onsets. Note however, that by itself, this property is not sufficient to guarantee that a rhythm will be a good timeline. If all the rhythm's onsets are contained in one half-circle of the cycle then they obviously do not contain antipodal onsets. For example, the rhythm with IOI's given by [1-1-1-1-1-1-1-9] has no antipodal pairs of onsets, and it is quite useless as a timeline. Rhythms that contain no antipodal pairs of onsets are said to possess the *rhythmic oddity* property. The ethnomusicologist Simha Arom coined this term after his discovery that most of the rhythm timelines used in the traditional music of the Aka Pygmies in Central Africa exhibited this property. Note that this property is invariant to rotations of the rhythm. In a cycle of sixteen pulses any consecutive inter-onset durations that sum up to eight pulses, will of course divide the cycle into two half-cycles. Therefore the rap, the shiko, and gahu rhythms in Figure 4 (as well as all their rotations) contain antipodal pairs of onsets, as does rhythm number 2. To obtain a more exciting rhythm timeline, those candidates that do not have the rhythmic oddity property may thus be discarded. However, that still leaves the son, rumba, bossa-nova, and the rhythms numbered 7 and 12 as contenders for the throne.

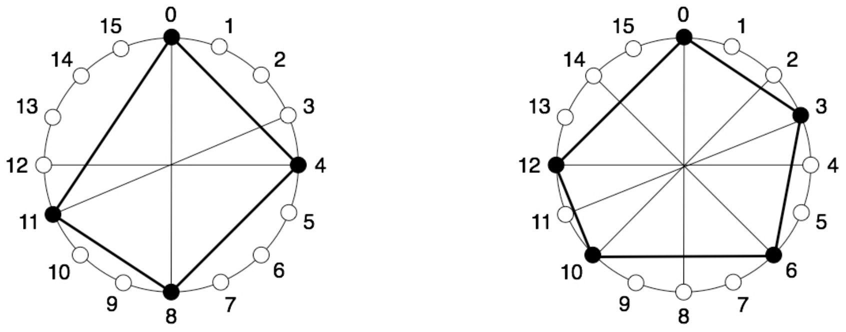


Fig. 5: The *conga* (left) does not have *rhythmic oddity* but the *clave son* (right) does.

THE SHADOW OF A RHYTHM

Picture yourself playing the 3-onset, 8-pulse rhythm with inter-onset-interval sequence [2-2-4], shown in Figure 6 (left) in polygon notation. While playing this rhythm observe the gesture of your arm (or hand, or mallet). In particular, focus on the distance between your arm and the instrument you are striking, as a function of time. Chances are that this distance function looks similar to the curve shown in Figure 7, which illustrates two cycles of the rhythm, and shows the points in time (pulses) at which your hand or other instrument strikes the instrument, namely 0,2,4,0,2,4,0.

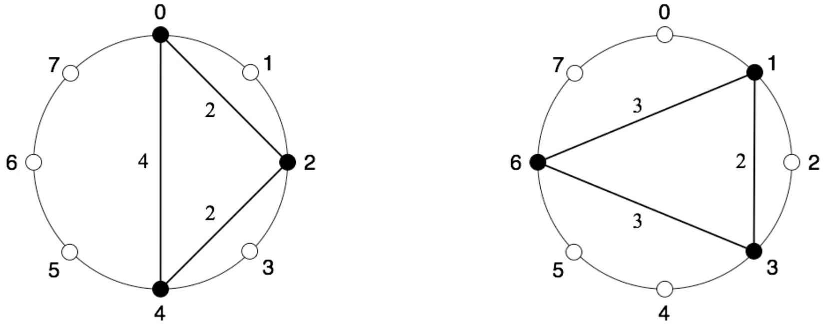


Fig. 6: The rhythm [2-2-4] and its *shadow*.

The points at which your arm achieves maximum height are likely to be the *midpoints* of the inter-onset-intervals, indicated in Figure 7 with vertical lines at pulses 1, 3, 6, 1, 3, 6. These midpoints themselves may be interpreted as determining another unsounded rhythm of sorts, a phantom of the rhythm actually heard. The resulting silent rhythm pictured in Figure 6 (right) is called the *shadow* of the rhythm [2-2-6] on the left. Since the muscles of the arm change their function at these midpoints in time, the nervous system must, perhaps unconsciously, register these moments in time. The performer, if not also the listener, must feel this phantom rhythm. Some musicologists therefore believe that shadow rhythms are physiologically and psychologically relevant to the proper study and understanding of rhythm. Indeed, some go as far as to claim that motion and motor action are essential for a satisfactory explanation of rhythm.

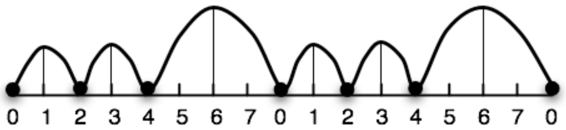


Fig. 7: The vertical motion of a hand, arm, or mallet as a function of time.

Shadow rhythms should not be confused with *subjective* rhythms. Subjective rhythms are those that are perceived by the listener, but not actually produced acoustically. That is, a subjective rhythm is an aural *illusion,* a perception that exists in the mind of the perceiver, but cannot be measured externally with scientific equipment, since it lacks a concomitant acoustic signal. This phenomenon is analogous to that of subjective contours in the domain of visual pattern perception, illustrated in Figure 8, where on the left we perceive the contours of a white triangle, and on the right those of a circle, both of which are not there. It should be emphasized that the subjective contours observed here are not *rational* conclusions about what must be there, but bona fide perceptions. In other words, with respect to the subjective triangle on the left, what is at stake is not the belief constructed from visual evidence that a white triangle is superimposed on three black disks, but rather that the non-existent lines themselves (the contours defining the boundary of an imaginary triangle) joining the pairs of disks, are perceived by the viewer. Similarly, with the figure on the right, the subjective contour does not refer to the fact that the viewer may conclude there is a white disk hiding a set of rays that meet at a central common point, but rather that a non-existent contour in the shape of a circle is perceived, a circle that has no visual signal that can be measured with photosensitive equipment. In both examples the white of the triangle and the disk also appears to be whiter than the white outside the triangle and disk.

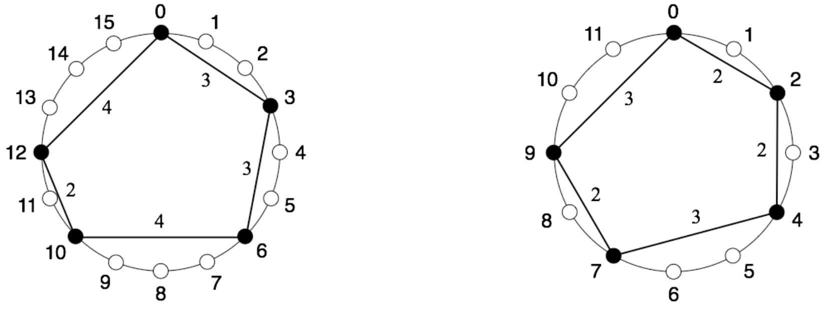
Fig. 8: Two examples of *subjective* contours in the visual domain.

Shadow rhythms and subjective rhythms should be distinguished from *inherent* rhythms, a term coined by the ethnomusicologist Gerhard Kubik. Inherent rhythms are those that are heard by the listener but not played by any single individual musician or instrument. Such rhythms may emerge from the interaction of different rhythms played on different instruments, or on the same instrument but with different tones, and unlike subjective rhythms they may exhibit acoustic reality, that is, they may be detected and measured with scientific equipment. This phenomenon is also called *streaming*.

If shadow rhythms are felt in some way by both musicians and listeners, even though they may not be sounded, and thus exhibit no acoustic reality, they must contribute in some way to the overall experience of the originating rhythms that shelter them. Furthermore, the nature of this contribution will be determined by the shared properties of the rhythm and its shadow. A useful notion for comparing a rhythm with its shadow, or more generally any two rhythms for that matter, is by means of *rhythmic contours.*

RHYTHMIC CONTOURS

Many applications in the field of music information retrieval require a method of measuring the similarity between two rhythms. Designing such a measure that agrees with how humans judge rhythm similarity is not an easy problem. Consider for example the two rhythms pictured in Figure 9. The clave son on the left is a *binary* rhythm defined on a cycle of sixteen pulses. On the other hand the rhythm on the right (sometimes called the fume-fume) is a *ternary* rhythm determined by a cycle of twelve pulses. Binary and ternary rhythms feel considerably different. Note that although both twelve and sixteen are even numbers, twelve is divisible without remainder by three, but sixteen is not. Objectively one would consider these two rhythms to be quite different from each other, yet many people, especially non-musicians, hear little or no difference between the two. There are good psychological reasons for this phenomenon. The human perceptual system has a tendency to register relative changes between elements of a sequence of time intervals better than their absolute values. In other words, it is easier for the brain to code the qualitative *kind* of change that occurs between two adjacent durations, than their exact (quantitative) values. More precisely, it is easier to tell whether the next interval in the sequence is shorter, longer, or the same as the previous interval, as compared with whether the next interval is exactly two-thirds, twice as long, or three times as long as the previous one. The clave son has inter-onset-intervals (IOI's) given by [3-3-4-2-4]. If we encode an increase in duration, a decrease in duration, and no change in duration by the symbols +, -, and 0, respectively, then the relative information for this rhythm can be encoded by the sequence of symbols [0, +, -, +, -]. This sequence is called the *rhythmic contour* of the rhythm. The fume-fume rhythm with IOI's given by [2-2-3-2-3] has exactly the same rhythmic contour as the clave son rhythm. Since the brain encodes the rhythmic contours of the two rhythms more faithfully than the exact IOI's, it is natural that many people should hear little difference between them. In other words, if two rhythms have the same contours, then from the perceptual point of view they may be considered to be very similar to each other.



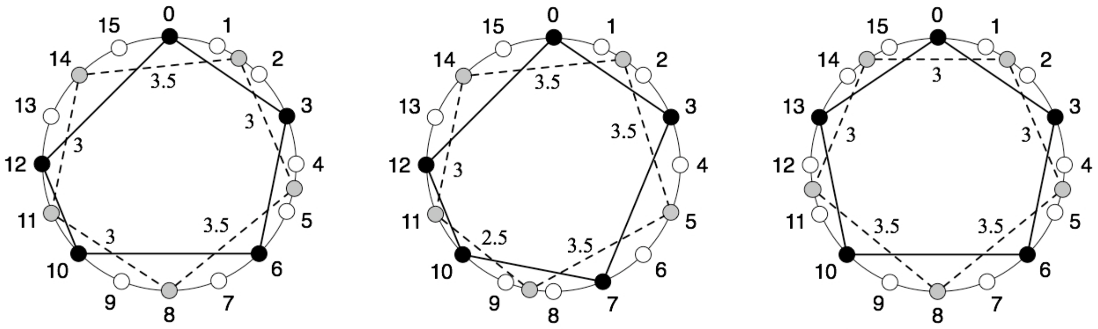
son

fume-fume

Fig. 9: Two different rhythms with the same *rhythmic contours*.

RHYTHM-SHADOW CONTOUR ISOMORPHISM

If one rhythm, say A, has a rhythmic contour that is a rotation of the rhythmic contour of another rhythm, say B, then the two rhythms A and B will be called *contour-isomorphic*. The son and fume-fume rhythms in Figure 9 constitute one example of a pair of contour-isomorphic rhythms. Thus a rhythm that is contour-isomorphic to its own shadow enjoys a privileged status among rhythms. Of the sixteen almost maximally even rhythms in Figure 4, the clave son is the only rhythm that is contour-isomorphic to its shadow rhythm. The six most distinguished of these are shown in Figure 10, along with their shadow rhythms and their inter-onset durations. The rhythmic contours of the rhythms and their shadows are shown in Figure 11. The rhythmic contour of the son is [0 + - + -] and that of its shadow is [+ - 0 + -] which is a rotation of the former.


son rumba bossa-nova

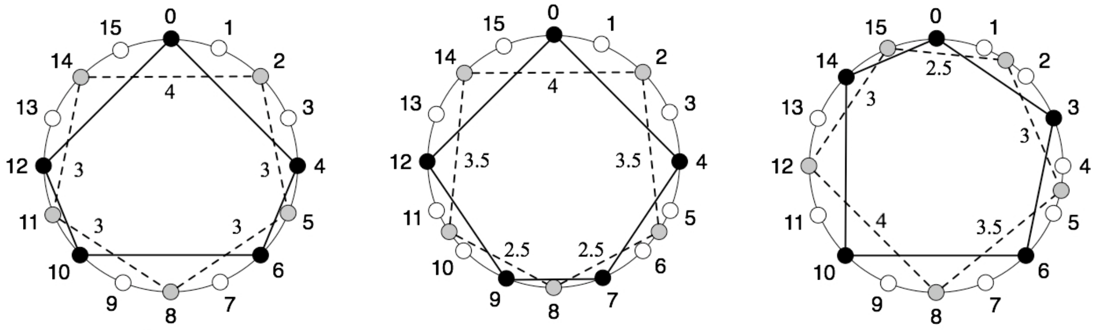

shiko rap gahu

Fig. 10: The six distinguished timelines and their *shadow* rhythms.

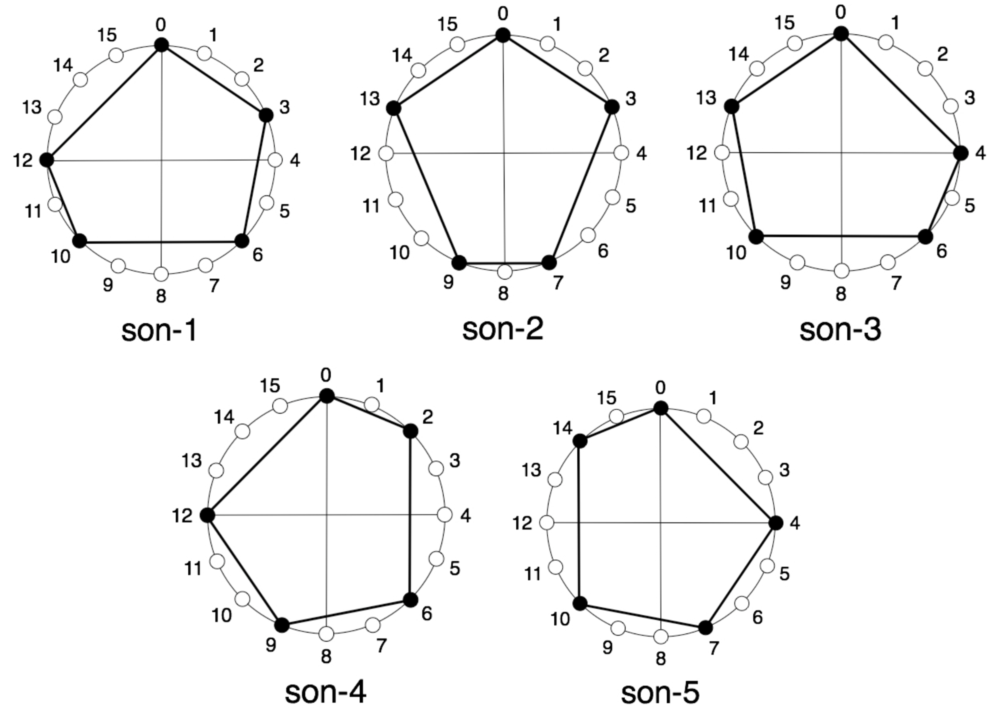
Timeline	Rhythm IOIs	Rhythm Contour	Shadow IOIs	Shadow Contour
Son	3, 3, 4, 2, 4	0 + - + -	3, 3.5, 3, 3, 3.5	+ - 0 + -
Rumba	3, 4, 3, 2, 4	+ - - + -	3.5, 3.5, 2.5, 3, 3.5	0 - + + 0
Bossa-Nova	3, 3, 4, 3, 3	0 + - 0 0	3, 3.5, 3.5, 3, 3	+ 0 - 0 0
Shiko	4, 2, 4, 2, 4	- + - + 0	3, 3, 3, 3, 4	0 0 0 + -
Rap	4, 3, 2, 3, 4	- - + + 0	3.5, 2.5, 2.5, 3.5, 4	- 0 + + -
Gahu	3, 3, 4, 4, 2	0 + 0 - +	3, 3.5, 4, 3, 2.5	+ + - - +

Fig. 11: The rhythmic contours of the six distinguished timelines, and their shadows.

METRIC DISSONANCE AND GESTALT DESPATIALIZATION

Although the rhythm-shadow-contour isomorphism property uniquely identifies the son rhythm from among the family of almost maximally even rhythms, one may wonder if starting the clave son at one of its other onsets might not result in a better timeline. Furthermore, the rhythm-shadow-contour isomorphism property does not uniquely characterize the clave son among its rotations because this property is invariant to rotations, and therefore holds for all rhythms that are rotations of the clave son. In this section it is demonstrated that with one additional property of good rhythms all rotations of the clave son may be eliminated from the competition. The son and its four rotations, labeled son-1 (no rotation) through son-5, are shown in Figure 12. The rhythm labeled son-2 indicates that this rhythm is the same as the son when the son is started on the second onset, and so on.

One of the most effective ways to add spice to rhythms by means of syncopation is to use it in such a way as to create what might be called a slight temporary confusion or cognitive insecurity, or metrical dissonance, or what Neil McLachlan calls a *gestalt despatialization*. In the case of 16-pulse timelines, which necessarily have four strong fundamental four-pulse beats felt at pulses 0, 4, 8, and 12, a gestalt despatialization can be introduced by first misguiding the listener into perceiving and cognitively predicting a sequence of three-pulse duration intervals. For this to happen there must be at least two initial IOI's of duration equal to three pulses. This means that the first three onsets must occur at pulses 0, 3, and 6. However this is not sufficient. To achieve the gestalt despatialization the last IOI of the rhythm must have duration equal to four units, and hence be determined by onsets at pulses 12 and 0. In this way the rhythm ends with two clear fundamental four-pulse beats.



son-1 son-2 son-3

son-4 son-5

Figure 12: All five rotations of the clave son timeline.

The stage is now set for inserting the fourth and last onset of the rhythm. One might be tempted to push the issue by continuing to insert three-element intervals for as long as possible until pulse number 12 is reached, as pictured in Figure 13. This would yield an optimal maximally even rhythm, number 16 in Figure 4. However, such a choice puts too much weight on IOI's of length three and too little weight on IOI's of length four (four versus one). It is not well balanced. Furthermore, at the end of the rhythm cycle the difference between the interval of length three and that of length four is not large enough to clearly indicate the change in underlying meter, and thus the despatialization effect is weakened. Therefore the onset at pulse number 9 in Figure 13 should be moved to one of its four neighboring positions at pulses 7, 8, 10, or 11. However, placing it at pulses 7 or 11 would create one very small interval of unit duration and a very long one of five units duration, violating the property of almost maximal evenness. Thus we are left with positions 8 and 9 at which to place this final onset. But inserting it at pulse number 8 would violate the rhythmic oddity property, since 8 is diametrically opposite to 0. Thus the only possible location for the fourth onset is at pulse number 9, yielding the clave son.

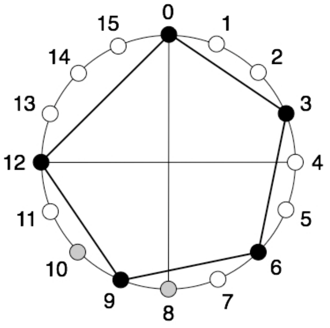


Fig. 13: The onset at pulse 9 can move only to either pulse 8 or pulse 10.

CONCLUDING REMARKS

The rhythmic pattern with inter-onset-intervals [3-3-4-2-4] is the rhythm that has conquered the world by becoming a universal rhythm. It has been called by different names in different places: the *Bo Diddley* beat in the United States, the *clave son* in Cuba, and the *kpanlogo* bell pattern in Ghana. The rhythm has been documented historically as early as the year 1258 by Safi al-Din who referred to it by the name *al-thaqil al-awwal*. More recently musicologists have celebrated its widespread saliency. Here it has been shown that this rhythm may be uniquely characterized by invoking several musicological properties of what makes a "good" rhythm good. These properties are: almost maximal evenness, rhythmic oddity, rhythm-shadow contour isomorphism, and gestalt despatialization. Interestingly enough, these properties are all mathematical in nature. It is surprising that with mathematics it is possible to explain the genius behind this irresistible rhythm.

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