

# Data Structures and Algorithms COMP-251 A

## Problem Assignment #2

### 1. Algorithms for Turing Machines

Let  $T(6)$  be a six-state Turing machine. Design an algorithm for  $T(6)$  that subtracts a number  $y$  from a number  $x$ . Your algorithm should work for any numbers that are positive integers of any finite size where  $x$  is greater than  $y$ . Assume the numbers are written on the tape in *unary* notation and separated by one blank. Use the symbols  $R$ ,  $L$ ,  $X$  and  $-$  for: move right, move left, make a mark and erase a mark, respectively. Assume the machine starts in state 1 and the reader is located on the leftmost mark on the tape. The machine should go to state 0 (stop) when finished. Let your instruction units (lines of code) be *4-tuples* of the form  $(a, A, B, b)$  where  $a$  denotes the present state,  $A$  takes on the symbols  $X$  or  $-$ ,  $B$  takes on the symbols  $R$  or  $L$  or  $X$  or  $-$ , and  $b$  denotes the next state entered.

### 2. Growth of Functions

Graph the functions  $12n$ ,  $6n \log n$ ,  $n^2$ ,  $n^3$ , and  $2^n$  using a logarithmic scale for the  $x$  and  $y$  axes. In other words, if the function value  $f(n)$  is  $y$ , plot this as a point with  $x$ -coordinate at  $\log n$  and  $y$ -coordinate at  $\log y$ .

### 3. Big “Oh” Notation

Problem 3.5 in the Udi Manber text.

### 4. Minimum Spanning Trees

Let  $S$  be a set of  $n > 2$  points in the plane in general position which is the union of two non-empty sets of points  $B$  (blue) and  $R$  (red). Prove (by any method of your choice) or disprove (by any method of your choice) that the minimum distance between a blue point and a red point determines an edge in the  $MST(S)$ , the *minimum spanning tree* of  $S$ . Points in general position means that no three points lie on a line.